TRINITY COLLEGE

**Semester One Examination, 2023**

**Question/Answer booklet**

# MATHEMATICS APPLICATIONS UNIT 3

**SOLUTIONS**

## Section One: Calculator-free

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

### Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

### Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 5 | 5 | 50 | 51 | 35 |
| Section Two: Calculator-assumed | 10 | 10 | 100 | 96 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**See next page**

**Section One: Calculator-free** **35% (52 Marks)**

This section has **five** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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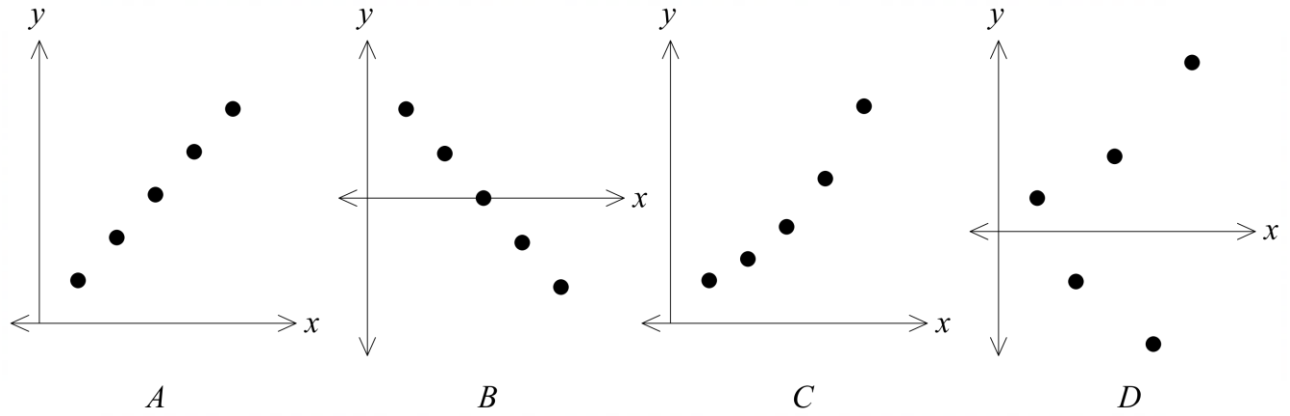
**Question 1** **(6 marks)**

Four graphs, $A$, $B$, $C$ and $D$ are drawn below.




State which of the graph, or graphs, above fit the following situations:

(a) There is a strong positive linear correlation between the two variables. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $A, C$ | ✓ States $A$.<br>✓ States $C$. | 3.1.6 |

(b) 100% of the variation in $y$ is explained by $x$. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $A, B$ | ✓ States $A$.<br>✓ States $B$. | 3.1.13 |

(c) This graph has a correlation coefficient closest to zero. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| $D$ | ✓ States $D$. | 3.1.7 |

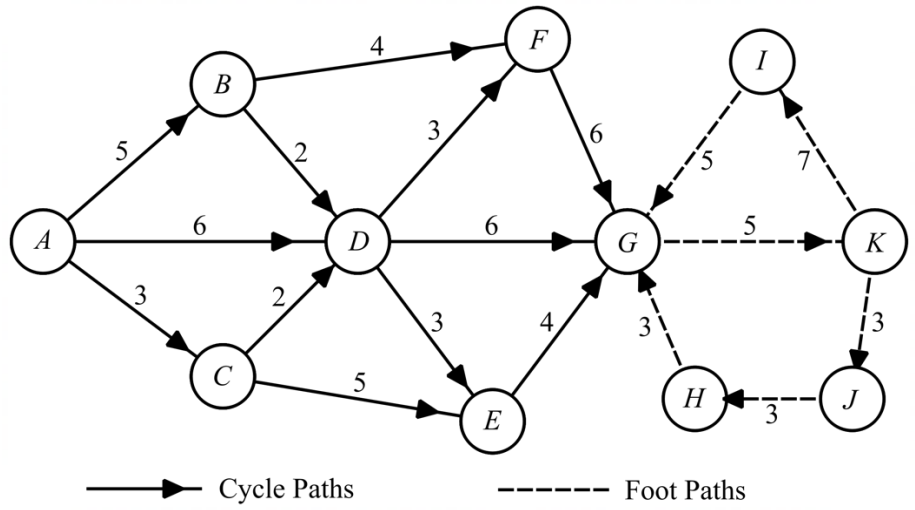
(d) The graph shows a sequence with a negative common ratio. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| $D$ | ✓ States $D$. | 3.2.6 |

**See next page**

**Question 2** **(12 marks)**

The network shows the cycle paths (indicated by a solid line) and footpaths (indicated by a dotted line) between various locations $A$ to $K$ in a city, and the time taken in minutes to travel along those paths.




George lives at $A$ and needs to cycle to work at $G$.

(a) (i) Determine the path he should ride along in order to get to work in the shortest possible time. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $ABFIG = 14$ minutes<br>$ADG = 12$ minutes<br>$ACDG = 11$ minutes<br><br>Quickest route is $ACDG$ | ✓ Uses an appropriate method to determine the shortest time.<br><br>✓ Determines the quickest route. | 3.3.7 |

(ii) State the time taken to ride this path. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| 11 minutes | ✓ States time that corresponds to quickest route found in part (a)(i). | 3.3.7 |

George notices that at each intersection ($B, C, D, E$ and $F$) he needs to wait, on average, 1.5 minutes for the traffic lights to change.

(b) Does the path George uses in part (a) still get him to work in the shortest possible time? Justify your answer by showing appropriate calculations, and stating any changes George should make. (3 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $ACDG$ is no longer the quickest<br>Time is now $11 + 2 \times 1.5 = 14$ minutes<br>$ADG$ is now the quickest<br>$12 + 1.5 = 13.5$ minutes | ✓ States route in part (a) is no longer optimal.<br>✓ Determines new route.<br>✓ Justifies selection of new route by calculating times. | 3.3.7 |

**See next page**

**Question 2** (continued)

When George arrives at work, he notices he has a flat tyre. He decides to walk to a bicycle repair shop at $K$, and then get coffee at either Inca Café (at $I$) or Java Coffee (at $J$), before returning to work at $G$.

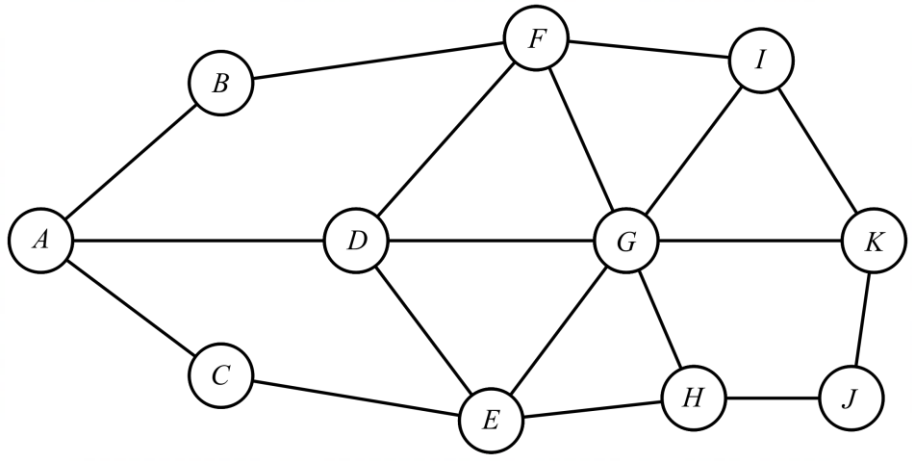
(c) (i) State the mathematical name of the route George will take. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| Cycle<br>(or closed path) | ✓ States cycle or closed path. | 3.3.6 |

(ii) Determine which coffee shop George should visit in order to complete these tasks in the shortest possible time. Justify your answer. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| George should go to Java Coffee<br>$GKIG = 17$ minutes<br>$GKJHG = 14$ minutes | ✓ States he should go to Java.<br>✓ Justifies choice of coffee shop. | 3.3.6<br>3.3.7 |

The network shows the roads between various locations, $A$ to $K$.




(d) Is the network semi-Eulerian? Justify your answer. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| No, as there are four vertices<br>of odd degree. | ✓ States no.<br>✓ Uses a relevant justification. | 3.3.8 |

(e) State an edge that would need to be added to make this network Eulerian. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| Edge must include two of the following verticies: $A, H, I$ or $K$ | ✓ States an edge. | 3.3.8 |

**Question 3** **(11 marks)**

A group of 10 students were asked to complete a multiple-choice quiz consisting of 10 questions.

The partially completed table below gives the number of hours of study completed by each student ($x$), the number of questions answered correctly (out of 10) ($y$), the predicted correct answers, and the residuals. Some of the data is missing.

| **Student** | Tim | Joon | Jim | Kylie | Steve | Anne | Cindy | Anton | Lukas | Marie |
|---|---|---|---|---|---|---|---|---|---|---|
| **Hours** ($x$) | 4 | 4.5 | 6 | 3.5 | 3 | 5 | 5.5 | 6.5 | 7 | 6.5 |
| **Correct Answers** ($y$) | 6 | 7 | | 6 | 5 | 7 | 8 | 8 | 9 | 9 |
| **Predicted correct answers** | | | | | | | | | | |
| **Residual** | $-0.1$ | 0.35 | 0.1 | 0.25 | | 0 | 0.55 | $-0.45$ | 0.2 | 0.50 |

The least squares regression line for the data was modelled by the linear equation

$$\hat{y} = 0.9x + 2.5$$

(a) Anne has a residual of 0. Explain what this means. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| Anne's mark was as predicted. | ✓ Explains. | 3.1.11 |

(b) Determine Steve's residual. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $\hat{y} = 0.9 \times 3 + 2.5 = 5.2$<br>$\text{Residual} = 5 - 5.2 = -0.2$ | ✓ Determines predicted score for Steve.<br>✓ Determines residual. | 3.1.11 |

(c) Determine how many questions Jim answered correctly. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $\hat{y} = 0.9 \times 6 + 2.5 = 7.9$<br>$0.1 = \text{Actual} - 7.9$<br>$\text{Actual} = 8$ | ✓ Determines predicted score for Jim.<br>✓ Determines actual score. | 3.1.11 |

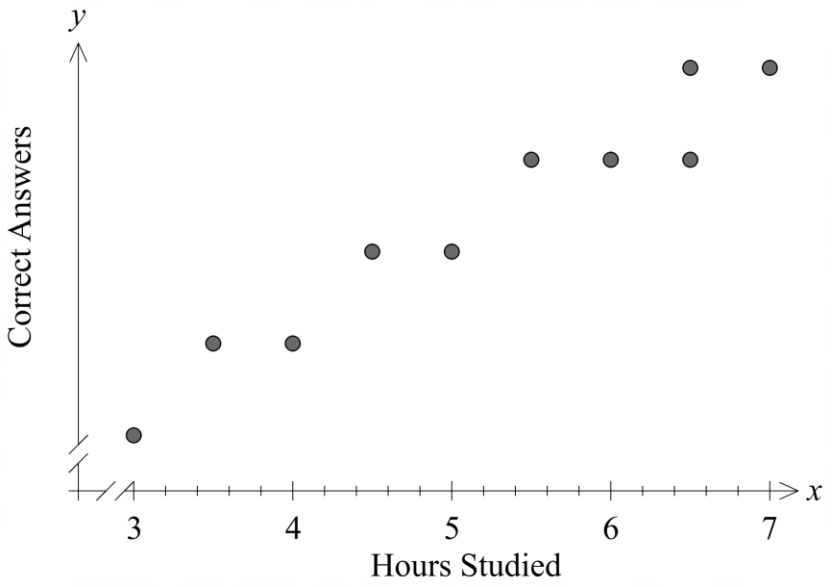
(d) How many students did better than predicted? Justify your answer. (2 marks)

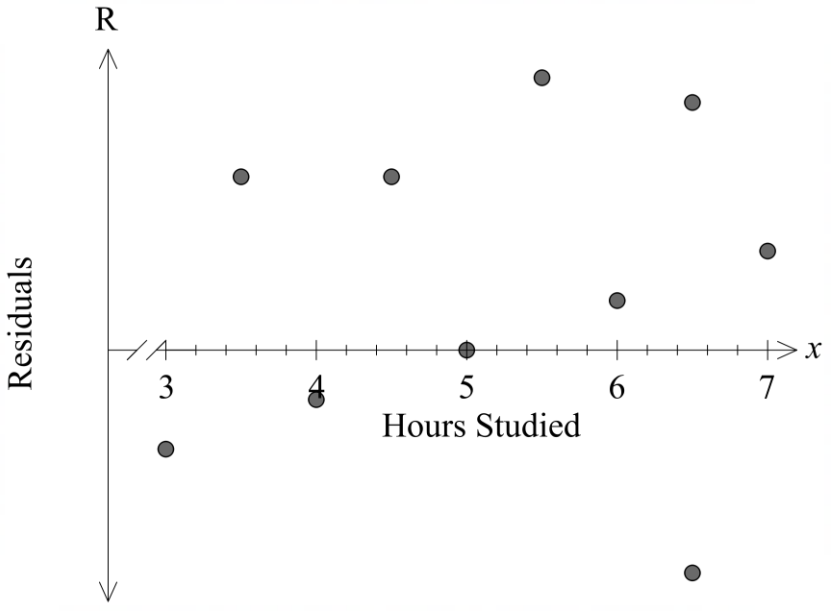
| Solution | Specific behaviours | Point |
|---|---|---|
| Six students did better than predicted.<br>These six have a positive residual. | ✓ States six did better than predicted.<br>✓ Explains why using the residuals. | 3.1.11 |

**See next page**

**Question 3** (continued)

A scatter plot of the data and a plot of the residuals are given below. The scale on the vertical axis has been removed on both of them.




(e) Describe the association between the hours studied, $x$, and the number of correct answers, $y$, in terms of direction and form. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| There is a positive linear association | ✓ States positive association.<br>✓ States association is linear. | 3.1.6 |

(f) Does the residual plot indicate that the data is well modelled by a linear equation. Explain your answer. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| Yes<br>There doesn't appear to be a pattern in the residuals. | ✓ States yes.<br>✓ States there is no discernible pattern in the residuals. | 3.1.11 |

**See next page**

**Question 4** **(12 marks)**

New Year 7 students at a school take part in a 'Greatest Race' event. In groups the students start at one of $A$, $B$, $C$ or $D$. They answer clues that give them their next destination. Students must return to their starting point.

The adjacency matrix $R$ (below) shows the order in which the students will travel.

$$R = \begin{array}{c} \\ \text{from} \end{array} \begin{array}{c} \begin{array}{cccc} \multicolumn{4}{c}{to} \\ A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

The $'0'$ in row $D$, column $A$ indicates that the clue at $D$ does not direct them to $A$.
The $'1'$ in row $D$, column $C$ indicates that the clue at $D$ directs them to $C$.

A group of students start at $A$, and must return to $A$.

(a) (i) Assuming they answer the clues correctly, state the route that the students take. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $ABDCDA$ | ✓ Route starts and ends at $A$.<br>✓ Route consistent with adjacency matrix. | 3.3.3 |

(ii) If a network was drawn for the adjacency matrix, $R$, explain why it must be a directed graph. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| The matrix is not symmetrical | ✓ States matrix is not symmetrical. | 3.3.3 |

(iii) Explain why any walks of length six in the network cannot be a trail. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| Any walk of length six must repeat a edge and trails cannot have any repeated edges. | ✓ States a walk of length six must repeat an edge.<br>✓ States a trail cannot repeat edges. | 3.3.6 |

**See next page**

**Question 4** (continued)

The adjacency matrix $W$ (below) shows the number of ways of travelling between the various destinations.

$$W = \begin{matrix} \\ A \\ B \\ C \\ D \end{matrix} \begin{matrix} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{matrix}$$

(b) Determine the number of possible closed paths that the students in part (a)(i) can take. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| 2 | ✓ States number of closed paths. | 3.3.2 |

(c) Give **two** reasons why the network represented by adjacency matrix, $W$, is not a simple graph. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| The network contains multiple edges<br>The network has a loop | ✓ States it has multiple edges.<br>✓ States in has a loop. | 3.3.1 |

The following matrices are calculated:

$$W^3 = \begin{bmatrix} 9 & 12 & 12 & 12 \\ 12 & 11 & 15 & 19 \\ 12 & 15 & 15 & 15 \\ 12 & 19 & 15 & 11 \end{bmatrix} \quad W + W^2 + W^3 = \begin{bmatrix} 12 & 16 & 16 & 16 \\ 16 & 17 & 20 & 23 \\ 16 & 20 & 20 & 20 \\ 16 & 23 & 20 & 17 \end{bmatrix}$$

(d) (i) Determine the number of walks of length 3 between $B$ and $D$. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| 19 | ✓ States answer. | 3.3.3 |

(ii) Describe why your answer to part (d)(i) does not tell you how many paths of length 3 there are between $B$ and $D$. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| Because some of the walks will repe<br>edges, e. g. $BDBD$. | ✓ States that some walks will repeat edges. | 3.3.6 |

(iii) Calculate the total number of walks of length 2 or 3 between $A$ and $C$. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $W + W^2 + W^3$ states there are 16 walks of length 1, 2 or 3<br><br>Hence, $16 - 1 = 15$ walks | ✓ Recognises there are 16 walks of length 1, 2 or 3.<br>✓ Concludes there are 15 walks of length 2 or 3. | 3.3.3 |

**See next page**

**Question 5** **(10 marks)**

A house is built near a river bank. Each year the river floods and erodes the river bank.

At the start of 2020, the house is $2.5$ m from the river bank. At the start of 2021, the river bank has eroded a total distance of 40 cm, and at the start of 2022 it has eroded a total distance of 60 cm.

(a) Assuming that the total distance eroded continues as an arithmetic sequence, determine

(i) the $n$th term of the sequence $D_n$, the total distance eroded $n$ years after 2020, in the form $D_n = a + bn$. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| In centimetres:<br>$D_n = 40 + (n-1)(20)$<br>$D_n = 20 + 20n$<br>In metres:<br>$D_n = 0.4 + (n-1)(0.2)$<br>$D_n = 0.2 + 0.2n$ | ✓ Substitutes in appropriate values into $n$th term formula.<br>✓ Expands and simplifies correctly. | 3.2.3 |

(ii) in which year the river bank will reach the house. (3 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $250 = 20 + 20n$<br>$230 = 20n$<br>$n = 11.5$<br>i.e. $2032$ | ✓ Substitutes in 2.5 or 250 into rule from part (a)(i).<br>✓ Solves for $n$.<br>✓ Rounds up and determines year. | 3.2.4 |

At the start of 2023, the river bank had eroded a total distance of $90$ cm.

(b) Explain why the arithmetic rule for $D_n$ cannot be used to model this sequence. (1 mark)

| Solution | Specific behaviours | Point |
|---|---|---|
| $90 - 60 = 30$ is not equal to the common difference | ✓ Explains why the arithmetic sequence is not appropriate. | 3.2.7 |

A geometric rule with a ratio of $1.5$ is required.

(c) (i) Determine a geometric recursive rule for $T_n$, the total distance eroded $n$ years after 2020 showing clearly how the ratio is calculated. (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $T_{n+1} = \frac{3}{2}T_n \quad T_1 = 40$ | ✓ Shows common ratio $r^2 = \sqrt{90/40}$.<br>✓ Determines recursive formula. | 3.2.7 |

(ii) In which year will the river bank now reach the house? (2 marks)

| Solution | Specific behaviours | Point |
|---|---|---|
| $T_4 = \frac{3}{2} \times 90 = 135$<br>$T_5 = \frac{3}{2} \times 135 = 202.5$ i.e. $2026$ | ✓ Determines fourth and fifth terms.<br>✓ Determines year when river bank will reach the house. | 3.2.8 |

**End of questions**

Supplementary page

Question number: _____